

Kinetics of Kapalbhathi: A mathematical interpretation of force, pressure, centre of gravity and centre of mass

Sonu Kumar, Shivam Gupta, Sandeep Singh, Nidheesh Yadav

Abstract— This article introduces the theoretical study of kinetics of kapalbhathi, how mathematical parameters correlated to each other. Kapalbhathi is high frequency yoga breathing (exhalation breathing rate up to 2 Hz) literally means shining forehead (kapala = cranium or forehead, bhathi = shining or even knowledge in Sanskrit). It is known widely for its positive effects on abdominal muscles and overall health. Kapalbhathi plays an important role to enhance the abdominal muscle strength and its muscle tone as well. The purpose of this theoretical study is to analyze or investigate the effects of pressure and forces at various sitting anatomical positions during kapalbhathi and also to find the exact location of centre of mass, centre of gravity at different positions of sitting postures. This article helps the practitioner to provide deep understanding and analysis for the mechanics of kapalbhathi as well as to reduce errors during practice. In this study a standard mathematical approach is adopted to find the relation between the force, angle and pressure. The study shows that the force which is exerted on the abdominal area during kapalbhathi pranayama is a mathematical function and how pressure changes as direction of force changes. This implies that as angle of sitting changes, it leads to increase force and it results to increase pressure ultimately and vice versa. There are significant changes in the outcome of the practice at different sitting body posture that leads to change in work of breath, heart rate and oxygen consumption. From the mathematical interpretation, we can conclude that practice of kapalbhathi pranayama at proper position gives a positive impact on the overall health.

Index Terms— Centre of gravity, Centre of mass, Force, Kapalbhathi, Kinetics, Pressure, Yoga

1 INTRODUCTION

YOGA techniques which involve conscious control of the breath with awareness and precision are called pranayamas. One such technique involves forceful exhalation and breathing at a high frequency approximately 1.0 Hz, though rates are high as 2.0 Hz are known. High frequency yoga breathing (HFYB) practice called kapalbhathi is yoga breathing technique characterized by forceful exhalation and high frequency breathing (exhalation breathing rate up to 2 Hz) literally means shining forehead (kapala = cranium or forehead, bhathi = shining or even knowledge in Sanskrit) (Telles et al., 2016). In Kapalbhathi Pranayama, the abdominal wall is brought into activity. This in turn works on the diaphragm and moves it better, thus helping the lungs empty efficiently. The minimum production of carbon dioxide during stillness and inactivity of physical muscles after kapalbhathi pranayama results in slowing the activity of the lungs and heart (Goel et al., 2013). Kapalbhathi as it is a complex form of yoga practice thus proper way of sitting during the practice

must be clear to practitioner. Literally, kapalbhathi means a yoga practice that makes the forehead shining. The word kapalbhathi is made up of two words kapal meaning skull (skull includes all the organs under the skull too) and bhathi means shining, illuminating.

Physiology of Kapalbhathi: Kapalbhathi is essentially a voluntary abdominal breathing. Here, Rechaka is active in nature and Puraka is passive in nature. So, Rechaka is more important part of this practice. Vertebral column automatically becomes straight making natural curve in the spine. Due to this position, the movement of the abdominal muscles is easily contracted during the practice of kapalbhathi. The vibrations or the mechanical jerk produced during vigorous practice of kapalbhathi are easily absorbed and counter balanced by this foundation. The force created by the abdominal muscles is properly directed up towards the chest.

Puraka: It is passive inhalation so that the abdomen protrudes out. Approx 100 to 200 ml air is inhaled in such a quick but passive and shallow inhalation. No frictional sound is produced in puraka as the air enters passively in the lungs without any force.

Rechak: It consists of active contraction of abdominal muscles and at the same time a forceful and rapid exhalation. Of course, the volume of axial air is same to that inhaled in Puraka. Little friction takes place at the opening parts of the nostrils, producing a sound just like one produces during moderate blowing of the nose. The standard frequency of breathing in kapalbhathi is 60 times per minute. The breathing is rhythmic and speed is acquired slowly by bringing proper coordination of the abdominal contraction and relaxation with

- Sonu Kumar is currently pursuing masters degree program in Yoga Sciences, University of Patanjali, India, PH-8497886656, E-mail: drsandeep.singh@gmail.com
- Shivam Gupta is currently pursuing Masters in Department of Yoga Science, University of Patanjali, India, PH-06260112411, E-mail: shivamkng2@gmail.com
- Sandeep Singh is currently working as an Assistant Professor of Yoga Research and Yoga Therapy in University of Patanjali, India, PH-08946870057, Email: drsandeep.singh@uop.edu.in
- Nidheesh Yadav is currently working as an Assistant Professor of Yoga Therapy and Ayurveda in University of Patanjali, India, PH-07302871717, Email: nidheesh@uop.edu.in

exhalation and inhalation.

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Mathematical Parameters

Force: A force is any interaction that, when an opposed, will change the motion of an object. A force can cause an object with mass to change its velocity which includes to begin moving from a state of rest i.e., to accelerate. A force can also be described in intuitively as a push or pull. It has both magnitude and direction making it a vector quantity. SI unit of force is Newton (N).

Pressure: It is defined as the amount of force applied at right angle to the surface of an object per unit area

Mathematically:

$$P = \text{Force} / \text{Area}$$

Thus it implies that

$$P \propto 1/A \tag{1}$$

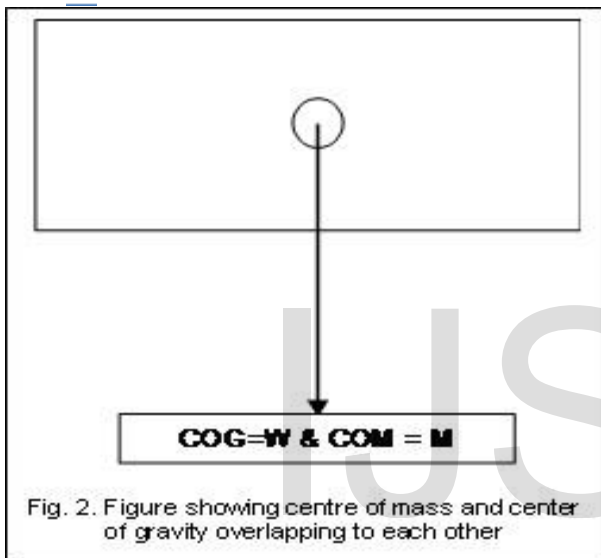


Fig. 2. Figure showing centre of mass and center of gravity overlapping to each other

Here F1, F2 and F3 are the three different functions of area. It depends on the nature of functions of area, pressure decreases either linearly or nonlinearly.

Centre of mass: It is defined as a point at which whole mass of the body or all the masses of a system of a particle appeared to be concentrated at one point.

Segmental method is to be used to calculate the centre of mass of human body as body is irregular in shape. Same method will be used to calculate the centre of mass of irregular shape of abdominal muscles involved during practice of kapalbhati. Since the shape of abdominal muscle is not geometrical. So, in order to calculate its area, concept of area of Irregular shape is used.

Centre of gravity: A point in an object where distribution of weight is equal in all direction and does depend on the gravitational field. In uniform gravitational field, COM and COG are overlapped to each other.

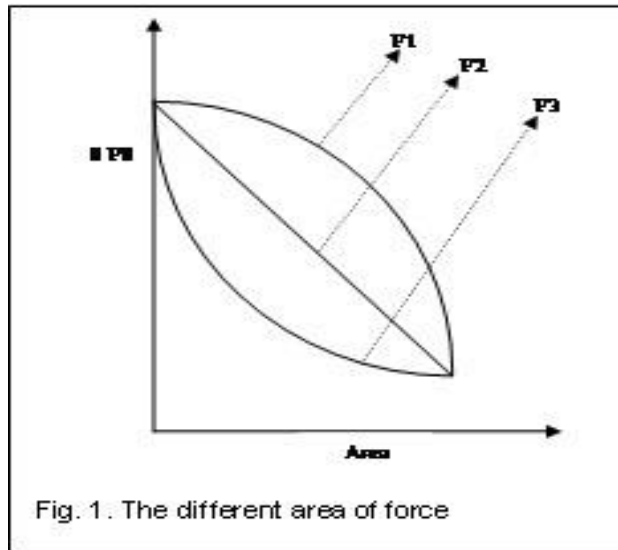


Fig. 1. The different area of force

Mathematical analysis / methodology

Mathematical analysis has been explained by considering three different cases:

Cases 1: when the body is in sitting posture making an angle of 90 degree right angle with respect to earth surface.

Let us consider a reaction force F by abdominal muscles which is equally and opposite in +X direction with respect to the external force exerted during Rechaka in - X direction. Let us consider centre of gravity is denoted by W, centre of mass M and pressure as P respectively. The direction of centre of gravity is downward to the Earth surface or -Y axis and magnitude is mg where m is equal to mass of the body and g is equal to acceleration due to gravity. Here, pressure force and centre of gravity are vector quantity. Here, in this case centre of mass is exactly lying on the origin of the co-ordinate axis. Due to uniform gravitational field, centre of mass and centre of gravity coincide each other.

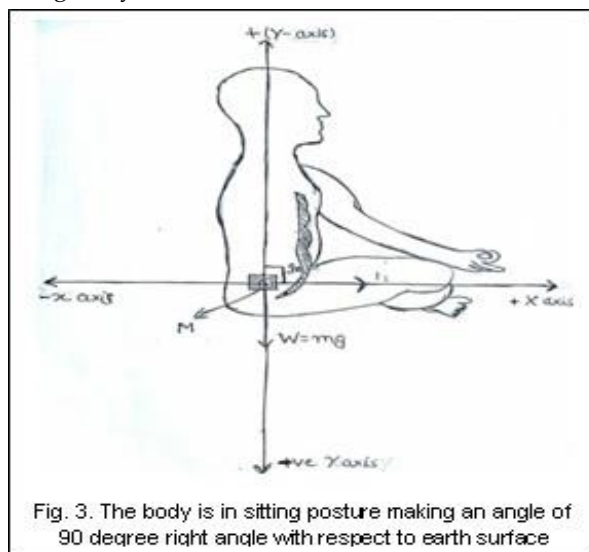


Fig. 3. The body is in sitting posture making an angle of 90 degree right angle with respect to earth surface

Mathematically

Thrust = Reaction Force = - External Force (F_{ext}) = - (-F) = F
 [By Newton’s third law of motion] (2)

Pressure (P) = Thrust / Area = - (F_{ext}) / Area (3)

Since F = γF sin θ (4)

P = F/A = γF sin θ / A (5)

Here, when θ is equal to 90° the external force is maximum this leads to the maximum abdominal reaction force by quick contraction gets an axial direction. This pushes the diaphragm upward into the thorax and it exerts an active maximum positive pressure on the lungs quickly. The blowing strength of diaphragm increases and air passage is cleared more effectively due to the forceful maximum air current. Here, centre of gravity is w = mg, centre of mass = M. Both centre of gravity and Centre of mass is laying at sacrum 2 (S2) points which is also the origin point of co-ordinate axes.

Cases 2: when the body is tilted forward making an angle of +θ° in clockwise direction with respect to its initial point. Here, in this case the reaction force (F) by abdominal muscles which is making an angle of θ with respect to initial position and + Y-axis. So this force vector F consists of two components one is in the + X direction and another is in the +Y direction respectively.

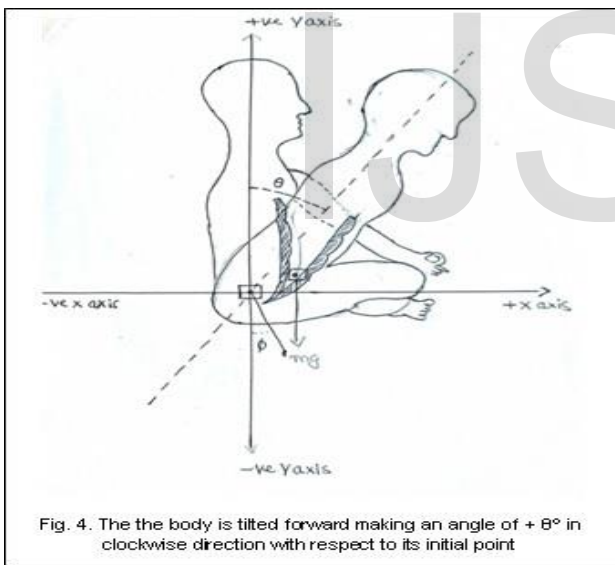


Fig. 4. The the body is tilted forward making an angle of +θ° in clockwise direction with respect to its initial point

Mathematically

Force F can be expressed as:

F = γF sinθ + γF cos θ (6)

Here, γF sin θ is mainly playing an important role for the production of air current in the air passage in axial direction. Fγ cos θ is vertical force responsible for increasing the vertical pressure.

So the pressure will be expressed as follows:

P = (γF sin θ + γF cos θ) / Area N/m² (7)

Let us take an example to elaborate this relation

Consider, θ = 45 degree

F = γF sin45 θ + γF cos45 θ = γF / √2 + γF / √2 (8)

F_{net} = √2 γF (9)

This implies that the external force exerting on abdominal area and the reaction force from the abdominal area is reduced

by 1/√2 times. But net external force is increased by √2 which implies that in this particular position pressure has increased by √2 times in forward direction due to the extra force component F cos θ in the vertical direction. This also shows that the blowing strength of diaphragm decreases by 1/√2 times and ultimately the cleansing of air passage are also reduced by 1/√2 times.

Cases 3: when the body is tilted backward making an angle of θ in anticlockwise direction with respect to its initial position. Here in this case reaction force (F) by abdominal muscles this is making an angle -θ with respect to initial position and positive y-axis.

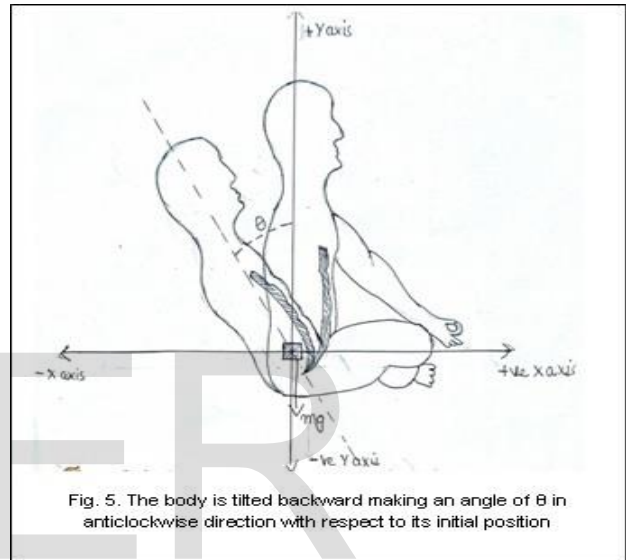


Fig. 5. The body is tilted backward making an angle of θ in anticlockwise direction with respect to its initial position

Mathematically

F = - γF sin θ + γF cos θ (10)

P□ = [- γF sin θ + γF cos θ] / Area (11)

Since P□ = Force/ Area

2 GRAPHICAL ANALYSIS

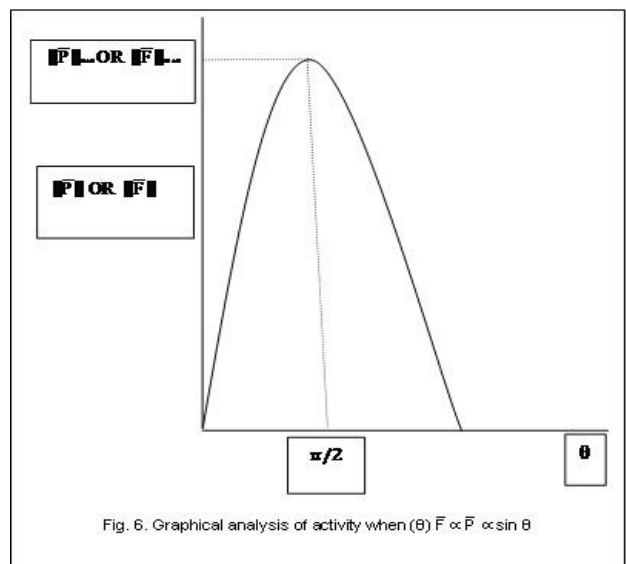


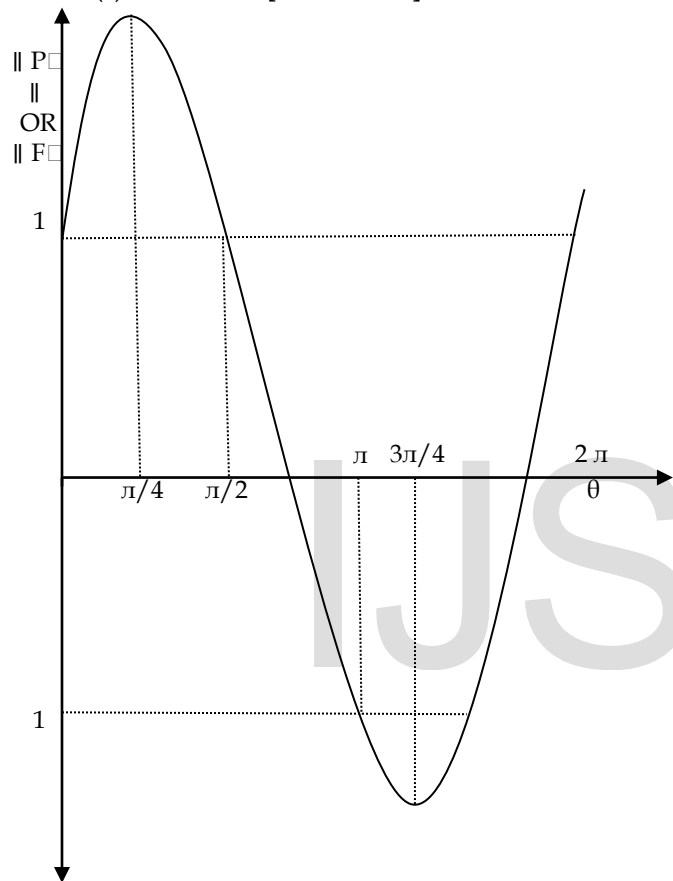
Fig. 6. Graphical analysis of activity when (θ) F ∝ P ∝ sin θ

Case 1: $(\theta) F \propto P \propto \sin \theta$

Discussion

As θ increases from 0 to $\pi/2 \rightarrow \sin \theta$ increases
 $\Rightarrow F \square$ increases and $P \square$ also increases up to $\pi/2$
 As θ increases from $\pi/2$ to $\pi \rightarrow \sin \theta$ decreases
 Hence, $F \square$ decreases and $P \square$ also decreases up to π

Case 2: $(\theta) F \square \propto P \square \propto [\sin \theta + \cos \theta]$



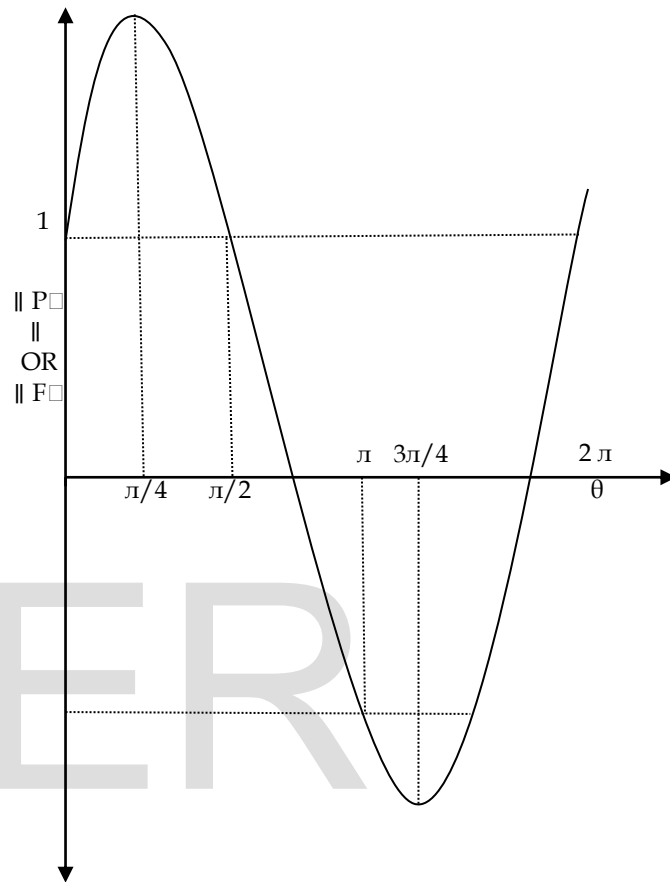
Discussion

As θ increases from 0 to $\pi/4 \rightarrow \sin \theta$ increases
 $\Rightarrow F \square$ increases and $P \square$ also increases up to $\pi/4$
 As θ increases from $\pi/4$ to $\pi \rightarrow \sin \theta$ decreases
 Hence, $F \square$ decreases and $P \square$ also decreases up to π
 As θ increases from π to $5\pi/4 \rightarrow \sin \theta$ reaches to negative maximum value
 $\Rightarrow F \square$ & $P \square$ also reaches up to negative maximum value i.e.;

-1.

As θ increases from $5\pi/4$ to $2\pi \rightarrow 2\pi \rightarrow \sin \theta$ increases
 $\Rightarrow F \square$ & $P \square$ also increases & reaches up to maximum value again i.e.; 1.

Case 3: $W \propto [\sin \theta + \cos \theta]$



Discussion

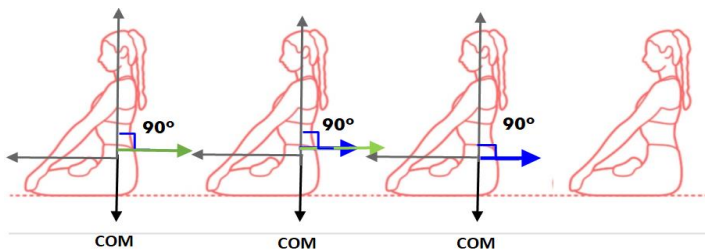
As θ increases from 0 to $\pi/4 \rightarrow W$ increases & reaches to + the maximum value at $\theta = \pi/4$
 As θ increases from $\pi/4$ to $\pi/2 \rightarrow W$ decreases & reaches to -1 value at $\theta = \pi$
 As θ increases from π to $5\pi/4 \rightarrow W$ decreases & reaches to negative maximum value at $\theta = 5\pi/4$
 As θ increases from $5\pi/4$ to $2\pi \rightarrow W$ increases & reach to again 1 at $\theta = 2\pi$

3 RESULTS

From the above mathematical interpretation we have calculated the centre of mass (COM) at different sitting position through which we can conclude major 3 points that:

Case 1: $(\theta) F \propto P \propto \sin \theta$

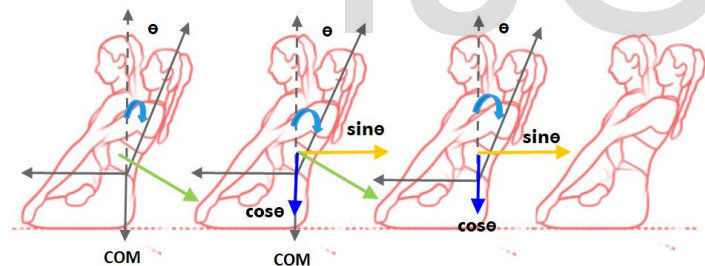
As we sit with spine erect making an angle of 90 degree with respect to the ground and practice then the pressure exerted direct only in horizontal direction thus effect the organ posterior to abdominal muscles involved in the practice.



When anyone practice kapalbhathi keeping the back erect then the pressure will direct only in horizontal region as vertical component ($\cos 90$) is zero and horizontal component ($\sin 90$) is maximum i.e. to one.

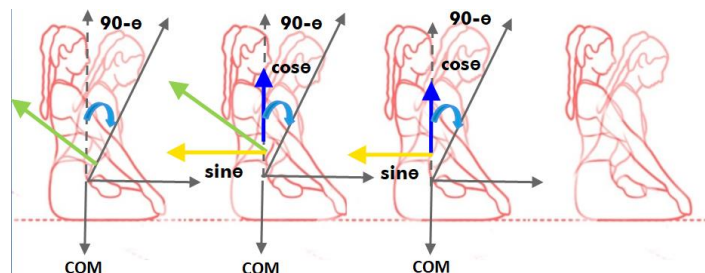
Case 2: $(\theta) F \propto P \propto [\sin \theta + \cos \theta]$

In the second case when the spine is tilted backward making a greater than 90 degree with respect to the ground then the vertical component the pressure exerted in downward direction during practice will affect the body.



Case 3: $W \propto [\sin \theta + \cos \theta]$

In the next case when the spine is tilted forward making an angle less than 90 degree with respect to the ground then the vertical component the pressure exerted in axial direction during practice will also adversely affect the body.



4 DISCUSSION

The purpose of this mathematical study is to reduce the ill effects of wrong practices of kapalbhathi as well as to explore the benefits of kapalbhathi for overall health. The major objective of the study is to calculate the centre of mass (COM) which is the base of any practice. You can retain in any of the body position if COM is balanced. Center of gravity (COG) which changes when body movement is done during the practice by which abnormalities generated from the wrong practices.

Another major objective is to displacement of pressure and rate of displacement of pressure during the practice of kapalbhathi at different angle of sitting (AOS).

5 FINDINGS

Calculation of centre of mass (COM)

As centre of mass is an imaginary point where whole mass of the any object can be considered to be concentrated. It can lie inside or outside the object as per the shape and size of the body. As human body is of irregular shape thus it is calculated by segmental method.

Location of COG changes as angle of sitting changes

AOS = 90 degree then horizontal pressure will be maximum.

AOS > 90 degree then displacement of pressure will start increasing in the downward direction.

AOS < 90 degree then displacement of the pressure will start increasing slowly in the upward direction. Pressure will increase gradually up to 45 degree then start decreasing slowly.

When AOS = 45 degree, the overall Pressure will be maximum. Here pressure in the vertical and horizontal direction will be of same magnitude.

6 END SECTIONS

6.1 Limitations

Kinetics of yogic practice is very fewer than any other field. Even till now apart from the mathematical modeling there are very less instrument available for the study of biomechanics.

Calculating the area abdominal muscles which is affected during the practice is difficult task because its shape is irregular and the effect varies with intensity, speed and time in which the practice is done.

Some of studies available is done only in one-dimensional and two-dimensional model of the asana. Mathematical modeling is the most accurate and effective method but it is very costly in nature as well as need great expertise.

6.2 Future implications

As it is mathematical study, I expect that in this particular topic more effective and empirical study should we done by the scholars in future.

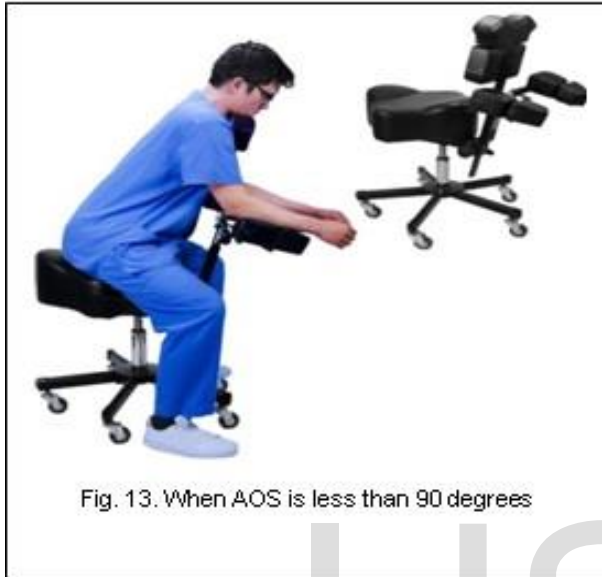
Study on larger sample on beginners and advance practitioners should be done.

Advanced equipment should be used for the data collection

and the analysis of the data.

From the result above we can say that work on supported kapalbhathi at different angle of sitting should be done with measurement of the effect on organ other than posterior to the abdominal muscles.

Supported kapalbhathi means to keep the back straight by the use of support as:



7 CONCLUSION

The mathematical study concluded that the practice of kapalbhathi with different position of sitting by keeping the vertebral column straight increases and decrease the pressure as in the axial direction upward and downward respectively.

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